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NONLINEAR ANALYSIS OF ELECTRO-THERMAL RESPONSE OF A CONDUCTING WIRE OF DISSIMILAR MATERIALS WITH VARIABLE THERMAL CONDUCTIVITY

S M Mahbobur Rahman, Anik Adhikary and S Reaz Ahmed

Department of Mechanical Engineering
Bangladesh University of Engineering & Technology, Dhaka-1000, Bangladesh

ABSTRACT

This paper presents a nonlinear analysis of the electro-thermal response of a conducting wire of dissimilar materials with variable thermal conductivity. The governing differential equation of the electro-thermal problem is derived considering the thermal conductivity of the material to be a function of both temperature and wire location. The solution of the nonlinear boundary-value problem is then obtained by converting it into an equivalent initial-value problem using a trial-and-error based iterative scheme together with the classical Runge-Kutta method. Electrical and thermal responses of a bi-metallic wire under the influence of a direct current flow are presented in details as a function of relevant parameters of interest. Results of the analysis are claimed to be highly accurate and reliable, and thus considered to be a valuable guide to the design of microelectronic devices.

Keywords: Nonlinear Analysis, Electro-thermal Problem, Bi-metallic Wire, Variable Thermal Conductivity.

1. INTRODUCTION

Accurate and reliable prediction of electro-thermal behavior of electrical conducting materials under different physical conditions is of utmost importance for the improved performance as well as integrity assessment of microelectronic devices. Now-a-days, it is of high practical importance to analyze these electro-thermal problems to determine the resultant temperature field properly. As a typical example of electro-thermal phenomenon, one can site electro-migration [1-2], which is the phenomenon of atomic diffusion due to current flow. When an electrical conducting material is subjected to a current flow, Joule heating is induced, which eventually leads to the generation of heat in the conductor. This electrical and thermal conduction ultimately causes thermal stress in the materials, which is considered to be one of the major reasons of metal line failure in electronic packaging.

The problem of heat conduction in a wire under the influence of current flow has been explained theoretically by Carslaw and Jaeger [3]. Steady temperature distribution near the tip of a crack in a homogeneous isotropic conductive plate was analyzed by Saka and Abe [4] under a direct current field with the help of path-independent integrals. Further, the analysis was extended by Sasagawa *et al*. [5] to determine the current density and temperature distributions near the corner of an angled metal line subjected to direct current flow. Greenwood and Williamson [6] treated the case of a conductor subjected to a direct current flow, in which temperature dependent material properties were considered, and showed that equipotentials were isothermals under the assumption that the relationship between the temperature and electrical potential at the positions of current input and output satisfied the condition of zero electro-thermal heat flux vector [7], and the remaining portion of the boundary was insulated both electrically and thermally. The method of Greenwood and Williamson was further extended by Jang *et al*. [8] to give a general solution to the coupled nonlinear problem of steady-state electrical and thermal conduction across an interface between two dissimilar half spaces. Wang *et al*. [9] also presented analytical solutions for the electrical and thermal conduction near the tip of a crack with a constant flux boundary condition at an infinite region. Recently, the method of Greenwood and Williamson was extended by Jang [10] to obtain a solution to the coupled nonlinear problem of steady-state electrical and thermal conduction across a crack in a conductive layer for which material properties were assumed to be functions of temperature. Very recently, introducing a new Joule heating residue vector, heat conduction in symmetrical electro-thermal problems has been analyzed under the influence of direct current passing through symmetrical regions of the boundary [7]. It has been shown that the Joule heating residue vector of symmetrical electro-thermal problem is related to the gradient of the temperature field associated with the problem without Joule heating.

This paper presents a nonlinear analysis of a heat conduction-convection problem coupled with an electrical problem subjected to a direct current flow. The thermal conductivity of the heat transfer problem is assumed to be a function of temperature and wire location. The flow of electricity through the wire causes a distribution of electrical potential over the length of the wire. The energy generation term in the governing equation is calculated based on the distribution of electrical potential, thereby leading to a coupled analysis of heat transfer problem with electrical problem. The solution of the nonlinear boundary-value problem is obtained by transforming it into an equivalent initial-value problem with the help of a trial-and-error based iterative scheme together with the classical Runge-Kutta method. The results of the analysis, especially, the distributions of temperature, electro-thermal heat flux vector, etc. for the case of a conducting wire of dissimilar materials (Cu-Al) are demonstrated mainly in the form of graphs. The influence of different relevant parameters of interest on the thermal behavior of the wire is also investigated.

2. MATHEMATICAL FORMULATION

2.1 Electrical Problem

Ohm's law for one dimensional potential distribution,

$$\frac{\partial \varphi}{\partial x} = -\rho J \tag{1}$$

The differential equation that governs the distribution of electric potential can be obtained by applying divergence operator on Eq. (1), which is

$$\frac{\partial^2 \varphi}{\partial x^2} = -\left(\frac{\partial \rho}{\partial x} J + \rho \frac{\partial J}{\partial x}\right) \tag{2}$$

For uniform wire and constant electrical resistivity the derivatives in the right hand side of the Eq. (2) can be neglected. Equation (2) is then reduced to the following Laplace's equation

$$\frac{\partial^2 \varphi}{\partial x^2} = 0 \tag{3}$$

2.2 Thermal Problem

The general governing equation for heat transfer in a conductive wire, the surface of which losses heat by convection to the surrounding atmosphere (T_∞) is

$$\frac{1}{A}\frac{\partial}{\partial x}(Ak\frac{\partial T}{\partial x}) - \frac{hC}{A}[T - T_\infty] + g = \alpha C_p \frac{\partial T}{\partial t} \tag{4}$$

For the present electro-thermal problem the internal heat generation per unit volume of the wire (g) is related to Joule heating caused by the current flow. For steady-state heat transfer in uniform wire with variable thermal conductivity $k(x,T)$, subjected to Joule heating, the governing equation reduces to

$$\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) - \frac{hC}{A}[T - T_\infty] + \frac{1}{\rho A_m}\left(\frac{\partial \varphi}{\partial x}\right)^2 = 0 \tag{5}$$

For the solution of electrical problem, the end conditions of the wire are simulated by the following relations

$$\frac{d\varphi}{dx} = \pm \rho \left(\frac{I}{A}\right) \tag{6}$$

The negative sign of the equation (6) applies to the wire end where current is being injected and the positive sign corresponds to the current outlet port. For the thermal problem, the temperatures at the two ends of the wire are assumed to be known. It is mentioned here that all possible physical conditions at the ends of wire can be successfully accommodated.

2.3 Electro-Thermal Heat Flux

Electro-thermal heat flux of a conductive wire is realized as a summation of the thermal heat flux and the flux representing the effect of electrical heating in the wire. The overall heat flux vector (P) related to the coupled electro-thermal problem, which is also known as the Joule heating residue vector [7], is defined by

$$P = -k \; grad\left[T + \frac{\phi^2}{2k\rho}\right] \tag{7}$$

Where, the first term in the right hand side of Eq. (7) is the thermal heat flux, $q = -k \; grad \, T$, and the second term reflects the corresponding effect of electrical heating.

3. METHOD OF SOLUTION

The present non-linear steady state heat

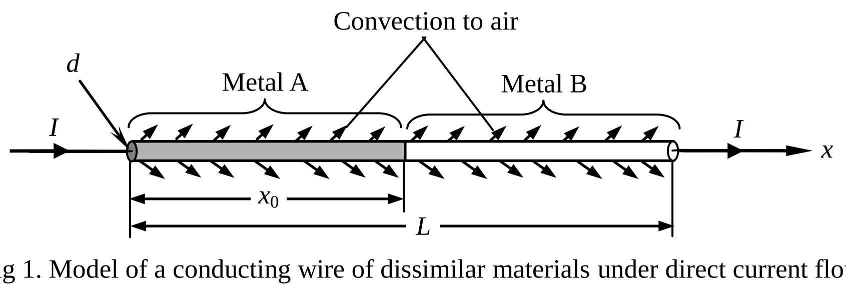

Fig 1. Model of a conducting wire of dissimilar materials under direct current flow

conduction-convection boundary value problem has been solved numerically considering it as an initial value problem. The boundary-value problem is converted to its equivalent initial value problem by the application of a trial-and-error based iteration scheme. Then the initial value problem is solved by the classical fourth order Runge-Kutta method. The boundary conditions at the both ends of non-linear problem are satisfied by an iterative fashion using the standard Bisection method. A MATLAB® based computer code was developed to solve the present problem. A total of 1500 nodal points have been considered to descritize the computational domain. The convergence as well as the stability of the numerical solution has been checked by varying the nodal points of the domain.

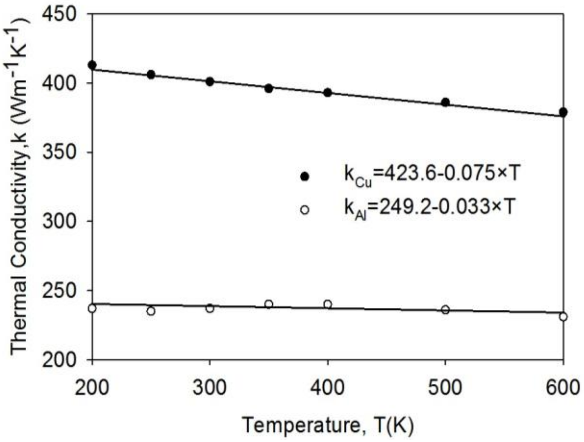

Fig 2. Thermal conductivity of Cu and Al as a function of temperature (200~600K)

The available data of thermal conductivity of the two metals, Copper and Aluminum for a temperature range of 200K-600K are shown graphically in figure 2 [11]. It is seen from the graph that thermal conductivity of Cu varies linearly with a small negative slope, while that of Al is found to be almost constant over the present temperature range. For the sake of present analysis, as observed from the available data, the thermal conductivities of both the materials are assumed to be linear functions of temperature.

The linear relationship of thermal conductivity of Cu and Al with temperature is approximated over the given temperature range according to the following equations,

$$k_{Cu} = a_1 + b_1 \times T = 423.6 - 0.075 \times T \quad (8)$$

$$k_{Al} = a_2 + b_2 \times T = 249.2 - 0.033 \times T \quad (9)$$

Then the Eq. (5) is reduced to the following governing equation of electro-thermal problem with variable thermal conductivity

$$(a_i + b_i T)\frac{\partial^2 T}{\partial x^2} + b_i\left(\frac{\partial T}{\partial x}\right)^2 - \frac{hC}{A}[T - T_\infty] + \frac{1}{\rho A_m}\left(\frac{\partial \varphi}{\partial x}\right)^2 = 0 \quad (10)$$

Here, i = 1, 2

The nonlinear governing differential Eq. (10) of the electro-thermal problem, which includes the influence of variable thermal conductivity, is thus developed by analyzing the available thermal conductivity data as a function of temperature for a particular range of interest.

Table 1: Electrical resistivity of Copper and Aluminum at room temperature (300K) [12]

Metal	Electrical resistivity (ρ) (Ω-m)
Copper	1.71×10^{-8}
Aluminum	2.65×10^{-8}

4. STATEMENT OF THE ELECTRO-THERMAL PROBLEM

A uniform conducting wire of dissimilar materials with diameter, d = 0.5mm is analyzed under the influence of constant direct current flow (I = 2A). The junction point of the bi-metallic wire is considered to be located at the mid-length position of the wire. At one end of the wire, current is injected and at the other end it goes out. The entire conducting wire (L = 0.15m) is assumed to be electrically insulated except for the two ends. Due to current injection there will be potential difference throughout the wire. Electric potential is related to current density. Current density can be calculated with the help of potential distribution. The boundary conditions for electrical problem are given in terms of prescribed current density.

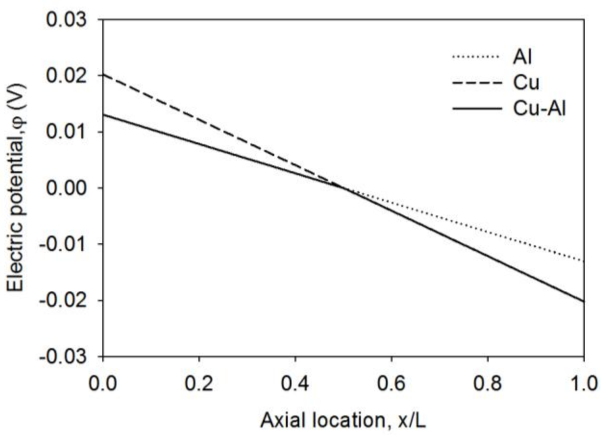

Fig 3. Distribution of electrical potential along the length of the metallic conducting wires (I = 2A)

As a result of current flow in the wire, Joule heating occurs. Volumetric internal heat generation is calculated with the help of potential distribution as shown in Eq. (5). The temperature distribution of the wire largely depends on thermal conductivities of the wire material. Thermal conductivity is assumed to be a function of temperature as well as wire location. The surface of the wire is assumed to transfer heat by convection to the surrounding environment which is at a temperature of

300K. The elevated temperature condition of the wire was simulated by assigning fixed temperature (318K) at the two ends of the wire. The convection heat transfer co-efficient is assumed to be constant (10 $Wm^{-2}K^{-1}$) for both the regions of copper and aluminum wires. Electrical resistivity of the two materials used (Cu & Al) are listed in Table 1.

5. RESULTS OF THE ANALYSIS

This section describes the results of the present nonlinear analysis of electro thermal responses of the bi-metallic conducting wire (Cu-Al). Figure 3 shows the distribution of electric potential, which varies linearly with the axial location of the conductive wires. Since all metallic conductors obey Ohm's law, it is obvious that the distribution of electric potential will be a straight line. But in case of the bi-metallic wire, the slope is different for each section of constituting metals, which is because of the fact that the properties are different for the two metals. As a result, sharp change of the slope is observed at the junction point of dissimilar materials.

Figure 4 describes the variation of the resulting temperature along the bi-metallic wire. In an attempt to compare the thermal response of the bi-metallic wire, the corresponding temperature distribution is presented together with those of the individual Al and Cu wires in figure 4(a). The temperatures are found to vary following a symmetric parabolic law for the Al and Cu wires, which is not the case for the bi-metallic wire. For Al and Cu wires, the highest temperature was found to occur at the mid-length position of the wires, but in case of bi-metallic wire (x_o/L=0.5), it is shifted from the mid-length position towards the right side or left side depending on the location of the junction (see figure 4(a)). The influence of the location of the junction point on the temperature distribution is illustrated in figure 4(b). The present analysis shows that the maximum temperature increases with the decrease of (x_0/L), that is, the relative length of the Cu wire decreases with respect to that of Al.

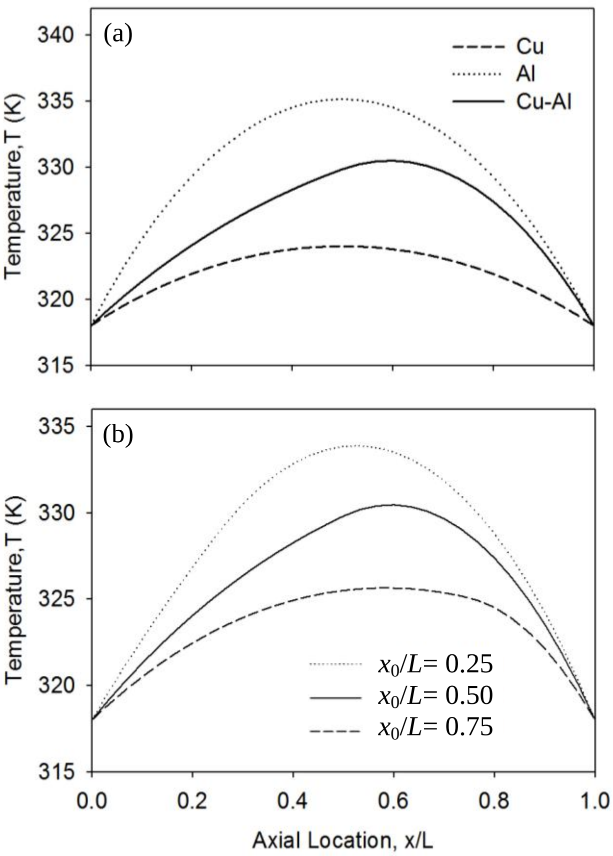

Fig. 4: Temperature distribution along the length of the wire with: (a) Cu-Al, x_0/L = 0.5, (b) different junction points of Cu-Al

A comparison of temperature distributions within the wire for the case of variable and constant thermal conductivity shows that the difference in maximum temperature is quite small (0.71^{o}C), which is because of the given temperature dependency of thermal conductivities of the metals for the present working temperature range, as shown in figure 2. However, the corresponding difference in temperature would be high if a different working temperature is considered, especially, very high or low temperature ranges. On the other hand, if the surface of the present bi-metallic wire is thermally insulated, the rise in temperature within the wire due to current flow is found to be nearly double of that observed with the bare wire.

Figure 5 shows the variation of electro-thermal heat flux along the length of wires. Nonlinear relationship is observed for all the three wires, which is because of the fact that electro-thermal heat flux is directly proportional to the gradient of square of electric potential, as shown in Eq. (7). The distribution of the heat flux along the length of the bi-metallic wire shows that the electro-thermal heat flux vector is lower in magnitude compared to those of the individual wires. The location of the junction point is however clearly reflected by the distribution of the heat flux for the bi-metallic wire. The overall magnitude of the present electro-thermal heat flux is found to be several orders higher when compared with that of thermal heat flux.

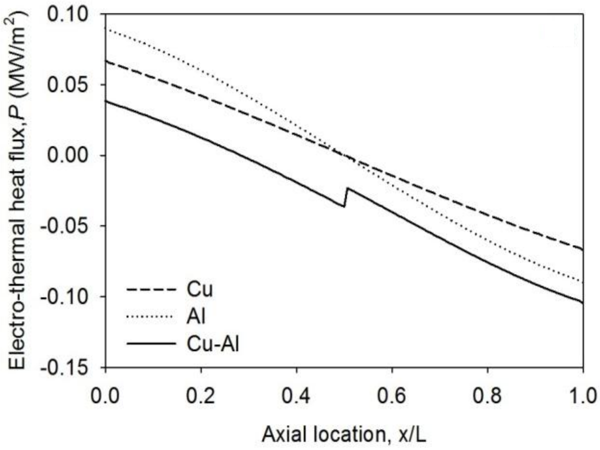

Fig 5. Distribution of electro-thermal heat flux along the length of the wires (x_0/L= 0.5, I = 2A)

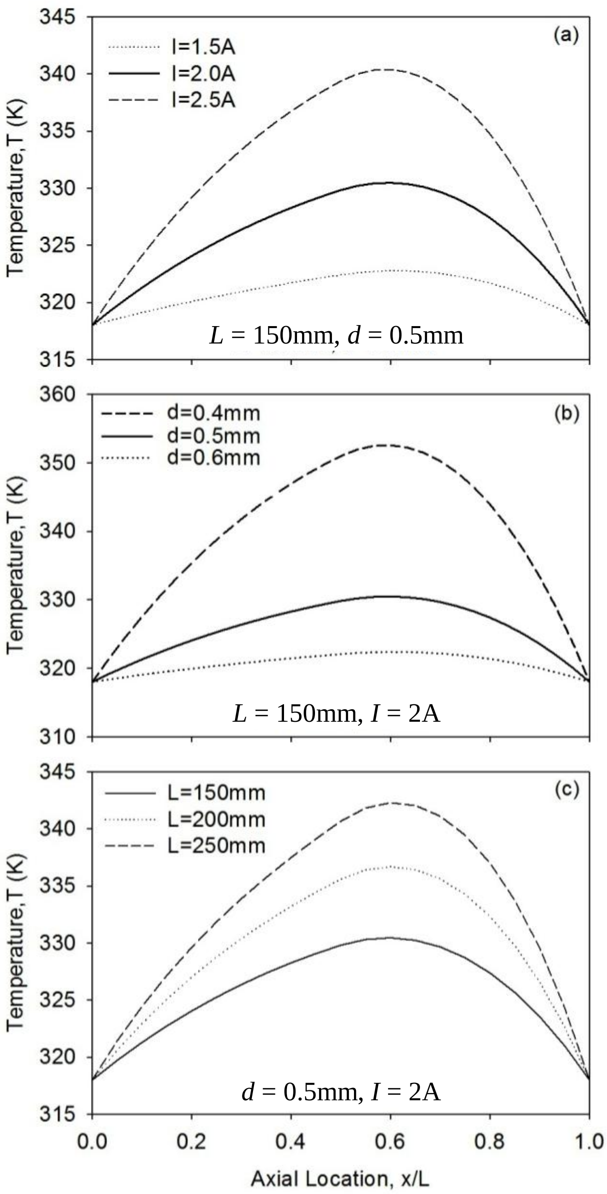

Fig 6. Influence of different relevant parameters on the thermal behavior of a Cu-Al bi-metallic wire

Finally, the influence of various parameters of interest on the thermal behaviour of the bi-metallic wire having the junction point, x_0/L=0.5 has been investigated, which is shown in figure 6. Figure 6(a) shows the influence of the amount of supplying current on the temperature distribution along the wire. For a uniform wire of fixed length, the maximum temperature within the wire is found to increase quite significantly as we increase the amount of current. Similar thermal response is observed, when the cross-sectional area of the wire is varied, while the wire length and supplied current are kept constant (see figure 6(b)). For example, the maximum temperature within the wire is found to assume values from 320 to 350K when the diameter was changed from 0.6 to 0.4 mm. Further, the temperature within the wire is found to increase with the increase of wire length, even though all other relevant parameters are kept constant in the analysis, the results of which are illustrated in figure 6(c). This is because, as the length of the wire is increased, the electrical resistance to current flow is increased, which eventually leads to higher electrical potential. This higher electrical potential causes the heat generation term in the governing equation to assume higher value, as the heat generation term maintains a nonlinear relationship with the electrical potential.

6. CONCLUSION

Electro-thermal response of a conducting wire of dissimilar materials has been analyzed taking into account the associated thermal conductivity as a function of temperature. The procedure of deriving the associated nonlinear governing differential equation of the electro-thermal problem has been outlined. It has been observed that the state of temperature as well as its distribution along the conducting wire of dissimilar materials differs significantly from those of the individual materials. The maximum temperature within the wire is also identified to be a function of the location of the junction point of the dissimilar materials. The influence of wire length, wire diameter and supplied current on the temperature rise is also found to be quite significant. Results of the analysis are claimed to be highly accurate and reliable and thus considered to be a valuable guide to performance evaluation as well as integrity assessment of metal lines used in modern microelectronic devices.

7. REFERENCES

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8. NOMENCLATURE

Symbol	Meaning	Unit
ρ	Electrical resistivity	Ωm
φ	Electric potential	V
J	Current density	Am^{-2}
k	Thermal conductivity	$Wm^{-1}K^{-1}$
h	Convective heat transfer co-efficient	$Wm^{-2}K^{-1}$
C_p	Specific heat capacity	$KJKg^{-1}{}^{\circ}C^{-1}$
P	Electro-thermal heat flux	Wm^{-2}
g	Volumetric internal heat generation	Jm^{-3}
C	Perimeter of wire	m
d	Diameter of wire	m
L	Length of wire	m
A_m	Mechanical equivalent of heat	$Jcal^{-1}$
q	Thermal heat flux	Wm^{-2}
A	Cross-sectional area of wire	m^2
T	Temperature	K
α	Material density	Kgm^{-3}

9. MAILING ADDRESS

S M Mahbobur Rahman
Department of Mechanical Engineering
Bangladesh University of Engineering & Technology, Dhaka-1000, Bangladesh
E-mail: mrahmanme@gmail.com